

Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level FURTHER MATHEMATICS

Paper 2

Exam Date

Morning

Time allowed: 2 hours

Materials

For this paper you must have:

- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

- 1 Given that $z_1 = 4e^{i\frac{\pi}{3}}$ and $z_2 = 2e^{i\frac{\pi}{4}}$
state the value of $\arg\left(\frac{z_1}{z_2}\right)$

Circle your answer.

[1 mark]

$$\frac{\pi}{12}$$

$$\frac{4}{3}$$

$$\frac{7\pi}{12}$$

$$2$$

- 2 Given that z is a complex number and that z^* is the complex conjugate of z

prove that $zz^* = |z|^2$

[3 marks]

The transformation T is defined by the matrix $\mathbf{M}$. The transformation S is defined by the matrix $\mathbf{M}^{-1}$. Given that the point (x, y) is invariant under transformation T , prove that (x, y) is also an invariant point under transformation S .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

-
- 4 Solve the equation $z^3 = i$, giving your answers in the form $e^{i\theta}$, where $-\pi < \theta \leq \pi$
[4 marks]

5 Find the smallest value θ of for which

$$(\cos \theta + i \sin \theta)^5 = \frac{1}{\sqrt{2}}(1 - i) \{ \theta \in \mathbb{R} : \theta > 0 \}$$

[4 marks]

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[5 marks]

[illegible]

The ball is to be modelled as a particle. The surface is assumed to be smooth and it is assumed that the forces applied to the ball by the springs are the only horizontal forces acting on the ball.

[5 marks]

7 (b)

Is the minimum distance predicted by the model likely to be too big or too small?

Explain your answer with reference to the model.

[2 marks]

[illegible]

8

show that $n I_n = (n - 1)I_{n-2} \quad n \geq 2$

[5 marks]

-
- 9** A student claims:
“Given any two non-zero square matrices, **A** and **B**, then $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ ”

- 9 (a)** Explain why the student’s claim is incorrect giving a counter example.

[2 marks]

- 9 (b)** Refine the student’s claim to make it fully correct.

[1 mark]

9 (c) Prove that your answer to part **(b)** is correct.

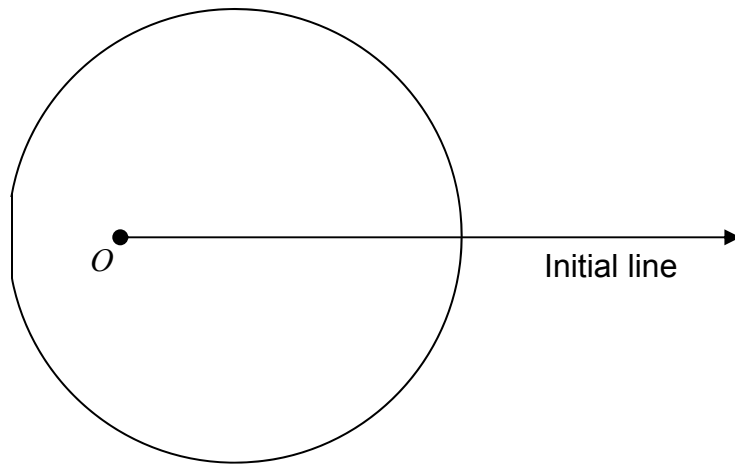
[3 marks]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Evaluate the improper integral $\int_0^{\infty} \frac{4x - 30}{(x^2 + 5)(3x + 2)} dx$, showing the limiting process used.

[8 marks]

[illegible]



The polar equation of C is $r = 4 + 2\cos\theta$, $-\pi \leq \theta \leq \pi$

[4 marks]

[illegible]

- Find the polar equation of the line segment AB

[illegible]

12

12 (a)

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

13 **S** is a singular matrix such that

$$\det \mathbf{S} = \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}$$

Express the possible values of x in terms of a and b .

[7 marks]

[illegible]

14 Given that the vectors **a** and **b** are perpendicular, prove that

$|(\mathbf{a} + 5\mathbf{b}) \times (\mathbf{a} - 4\mathbf{b})| = k|\mathbf{a}||\mathbf{b}|$, where k is an integer to be found.

Explicitly state any properties of the vector product that you use within your proof.

[9 marks]

[illegible]

[3 marks]

[illegible]

[2 marks]

15 (c) Hence show that $\sum_{n=1}^{\infty} \frac{1}{4^{n-1}} \cos 2n\theta = \frac{16\cos 2\theta - 4}{17 - 8\cos 2\theta}$

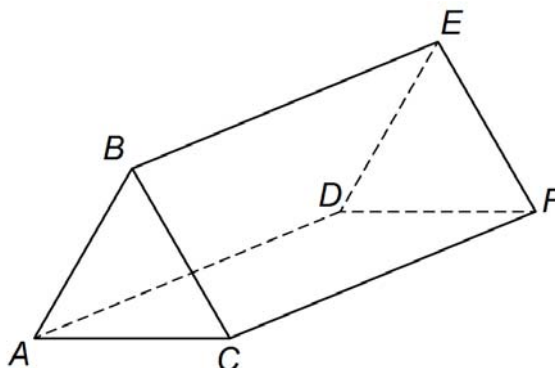
[4 marks]

15 (d) Deduce a similar expression for $\sum_{n=1}^{\infty} \frac{1}{4^{n-1}} \sin 2n\theta$

[1 mark]

16

A designer is using a computer aided design system to design part of a building. He models part of a roof as a triangular prism $ABCDEF$ with parallel triangular ends ABC and DEF , and a rectangular base $ACFD$. He uses the metre as the unit of length.



The coordinates of B , C and D are $(3, 1, 11)$, $(9, 3, 4)$ and $(-4, 12, 4)$ respectively.

He uses the equation $x - 3y = 0$ for the plane ABC .

He uses $\left[\mathbf{r} - \begin{pmatrix} -4 \\ 12 \\ 4 \end{pmatrix} \right] \times \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ for the equation of the line AD .

Find the volume of the space enclosed inside this section of the roof.

[9 marks]
